

1. Consider the *array code* \mathcal{C} built from \mathcal{H}_7 as follows:

- Arrange 16 information bits into a 4×4 matrix.
- Apply the \mathcal{H}_7 encoder to each of the rows to create a 4×7 array.
- Then apply the \mathcal{H}_7 encoder to each of the columns in the 4×7 array to create a new 7×7 array.
- \mathcal{C} consists of all 7×7 matrices formed in this way.

- (a) Determine the minimum Hamming distance d_{\min} for \mathcal{C} .
- (b) A code is said to be (τ_c, τ_d) -error correcting/detecting if it can correct every error pattern of Hamming weight at most τ_c and then, if failing to correct the error pattern, detect every error pattern of Hamming weight at most τ_d . Determine the set of all parameters τ_c and τ_d for which \mathcal{C} is a (τ_c, τ_d) -error correcting/detecting code and τ_d is as large as possible for a given choice of τ_c .

2. For each code length from $n = 2$ through 12, provide a table of systematic linear block codes of dimension $k = 2$ with best possible minimum Hamming distance d_{\min} . The table is to include a generator matrix for the code as well as the d_{\min} value.

3. Consider the $[4,10]$ binary code \mathcal{C} with generator matrix

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

The encoding rule $\bar{c} = (a_1, a_2, a_3, a_4)\mathbf{G}$ is not systematic. Find a matrix operator \mathbf{J} that maps code words (out of a decoder) to the corresponding information bits (for the end user). That is,

$$(a_1, a_2, a_3, a_4) = \bar{c}\mathbf{J}.$$

4. Let \mathcal{C} admit the following optimal (minimum distance) decoder:

Syndrome	Error Pattern
$[0000]^T$	0000000
$[1000]^T$	0001000
$[0100]^T$	0000100
$[1100]^T$	0001100
$[0010]^T$	0000010
$[1010]^T$	0001010
$[0110]^T$	0000110
$[1110]^T$	0111000
$[0001]^T$	0000001
$[1001]^T$	0010100
$[0101]^T$	0011000
$[1101]^T$	0010000
$[0011]^T$	0000011
$[1011]^T$	0100000
$[0111]^T$	1000000
$[1111]^T$	0010010

- (a) Determine a generator matrix for \mathcal{C} .
- (b) Use the decoder to decode the received vector $\bar{r} = (1110111)$.