JHU EE525.707 HOMEWORK #3 **©2007 Hammons**

1. Use the Sphere Packing and Gilbert bounds to give upper and lower limits on the required redundancy for a binary code with parameters $n = 23$ and $t_c = 3$. How does the binary Golay code compare to the bounds?

2. Let $\mathcal{C}$ be an $[n, k]$ linear code used for error detection over a BSC with crossover probability p. Using the MacWilliams Identity, derive a formula for the undetected word error probability $P_u(E)$ using the weight enumerator $\mathcal{B}(x)$ of the dual code $\mathcal{C}^{\perp}$.

3. Consider the [7,3] linear block code $\mathcal{C}$ whose systematic generator matrix is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

It is to be used to transmit systematically-encoded information over a BSC with crossover probability p. Use the standard array for $\mathcal{C}$, which is given below, to answer the following questions regarding decoding performance.

Standard Array

0000000	1000111	0101011	0011101	1101100	1011010	0110110	1110001
0000001	1000110	0101010	0011100	1101101	1011011	0110111	1110000
0000010	1000101	0101001	0011111	1101110	1011000	0110100	1110011
0000100	1000011	0101111	0011001	1101000	1011110	0110010	1110101
0001000	1001111	0100011	0010101	1100100	1010010	0111110	1111001
0010000	1010111	0111011	0001101	1111100	1001010	0100110	1100001
0100000	1100111	0001011	0111101	1001100	1111010	0010110	1010001
1000000	0000111	1101011	1011101	0101100	0011010	1110110	0110001
0000011	1000100	0101000	0011110	1101111	1011001	0110101	1110010
0000110	1000001	0101101	0011011	1101010	1011100	0110000	1110111
0001100	1001011	0100111	0010001	1100000	1010110	0111010	1111101
0011000	1011111	0110011	0000101	1110100	1000010	0101110	1101001
0001010	1001101	0100001	0010111	1100110	1010000	0111100	1111011
0010100	1010011	0111111	0001001	1111000	1001110	0100010	1100101
0010010	1010101	0111001	0001111	1111110	1001000	0100100	1100011
0111000	1111111	0010011	0100101	1010100	1100010	0001110	1001001

   (a) Determine the probability $P_U(E)$ of undetected word error.

   (b) Determine the probability $P(E)$ that the optimal minimum distance decoder will produce the wrong code word (decoder error).

   (c) Determine the probability that the optimal minimum distance decoder will decode all three information bits incorrectly.

   (d) Determine the probability $P(F)$ that a $t = 1$ bounded distance decoder gives up and does not produce an answer (decoder failure).

4. Examination of the standard array given in the previous problem shows that certain arbitrary choices were made among the coset leaders (“correctable” error patterns) in rows 9 and below. (The different possibilities correspond to differences in the ordering of like-weighted elements in the scratch list from which the standard array was built.)

   (a) In the last row, suppose that 1001001 was chosen instead of 0111000 as the coset leader. What will be the last row of the new standard array? Compute $P(E)$ for the new standard array decoder.

   (b) Repeat (a) but using 1111111 instead of 0111000 as the coset leader. (Note that this choice does not lead to a standard array! Why?)