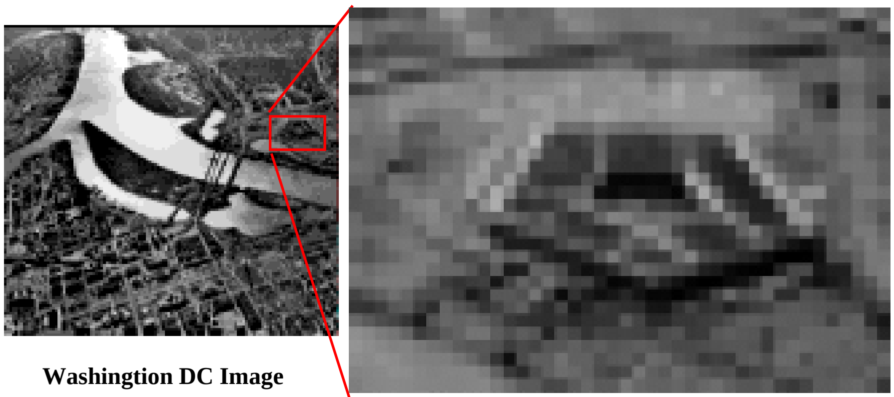
IMAGE COMPRESSION AND PACKET VIDEO

DC Image



Washingtion DC Image
Scaled 10X

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Data Profile

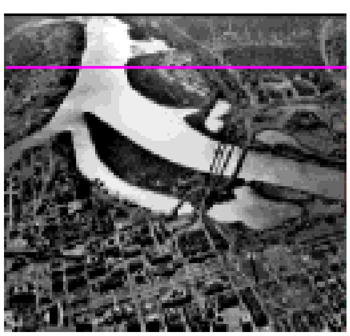
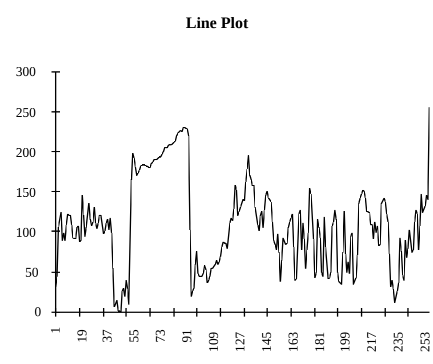


IMAGE: DC.img



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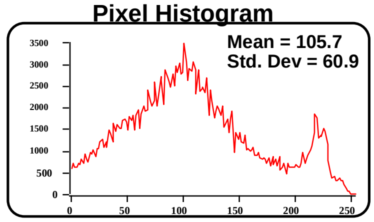
Difference Image Statistics

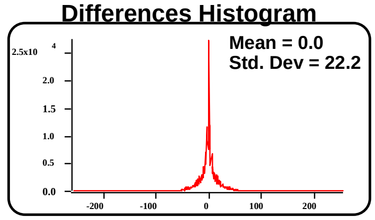
- Look at the histogram of the difference image

Original 'DC' Image

Pixel Histogram



Differences Histogram



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Histogram Statistics

- The histogram of the difference signal is highly peaked at origin which can be exploited by the Huffman coder.
- If we look at the histogram of a typical picture, it would be roughly a gaussian distribution.
  - Difference Histogram is roughly a Laplacian distribution

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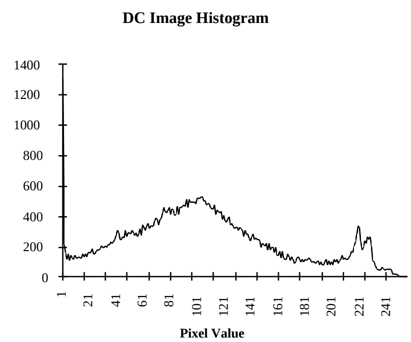
Image Histogram

IMAGE: DC.img



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Predictive Coding

- Two predictive coders are discussed in the literature (e.g. Anil Jain Fundamentals of Image Processing)
  – Differential Pulse Code Modulation (DPCM)
  – Delta Modulation (DM)
- Predictive Coders are among the simplest of lossless and lossy coders
  – Hardware requirements are minimal and have been built using older technology.

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Predictive Coding

- Basic idea: Predictive coding exploits the spatial correlation between adjacent pels (previously coded information) to make an approximate prediction of the sample to be encoded.
  - The differential signal resulting from the subtraction of the prediction from the actual value of the pel is quantized into a set of L discrete amplitude levels (lossy mode).
  - These levels could then be Huffman encoded.

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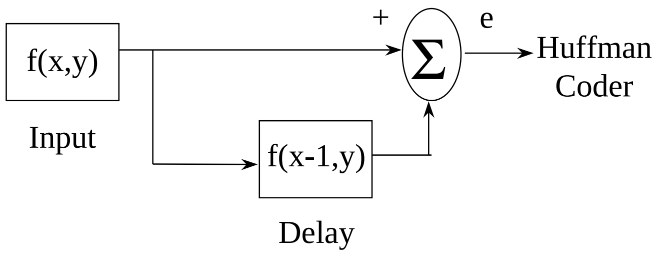
Predictive Coder

- A Lossy predictive encoder consists of two required, and one optional component:
  - Predictor; linear predictors, adaptive predictors
  - Quantizer; uniform, non-uniform, adaptive quantizer
  - Entropy encoder (optional); Huffman coder, arithmetic coder, rice coder (NASA standard)

Differential Coder

- Consider the coder shown below:



- A simple coder is designed by obtaining the differences in gray level of adjacent pixels

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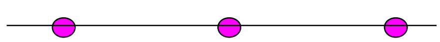
Differential Coder

- Original Pixel Values

  f(x-2,y) f(x-1,y) f(x,y) One line of a picture

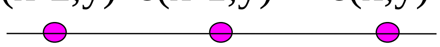
  $0 \le f(x, y) \le 255$



- Difference signal e(x,y) = f(x,y) - f(x-1,y)

  e(x-2,y) e(x-1,y) e(x,y) The error signal

  $-256 \le e(x, y) \le 255$



e(x,y) = f(x,y) - f(x-1,y)
e(x-1,y) = f(x-1,y) - f(x-2,y)

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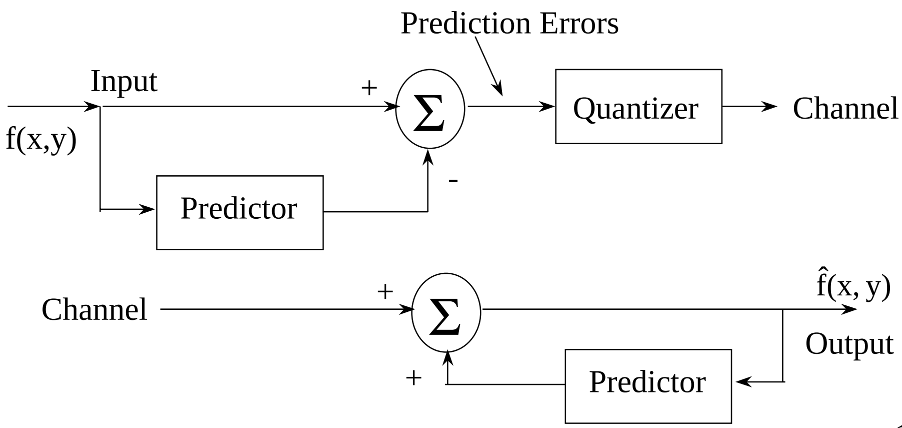
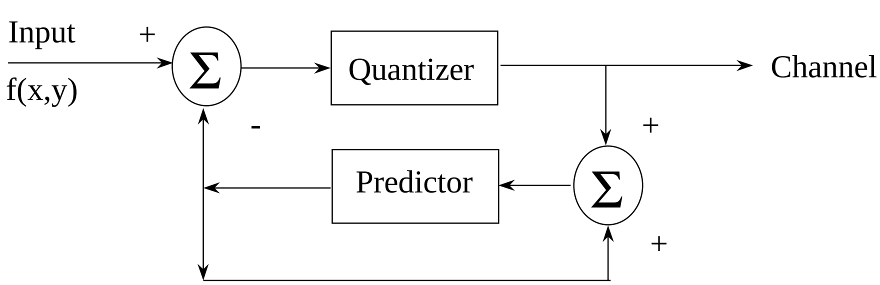
Feed Forward Differential Coder System



Differential Pulse Code Modulation (DPCM)

- Problem: Quantization Error Propagation
- Solution: Quantization Error Feedback



DPCM Coding

- In DPCM coding, the analog signal is sampled at the Nyquist rate
  - The predictor makes a prediction using the previously transmitted pixels for the current frame (Intraframe Prediction)
  - The predictor could use pixels from the previous fields (Intraframe Prediction)
    - » Intraframe Prediction requires previous frame storage.
  - Predictors using pels from the same scan line are called 1-D predictors, from the previous scan lines are called 2-D predictors

Linear Predictors

- First-Order Predictor:

$$\hat{x}_i = \rho x_{i-1} + (1-\rho)\mu$$

$$\rho = \text{CORRELATIONCOEFFICIENT}$$

$$0.85 \le \rho \le 0.95$$

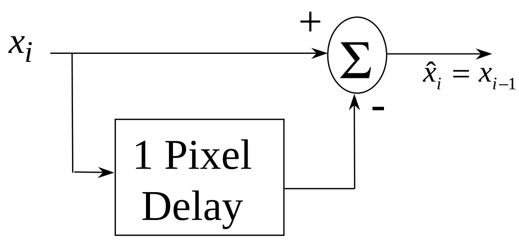
$$\mu = E[x] = \text{mean}$$

$$\text{MINIMIZES} \quad E\left[(x_i - \hat{x}_i)^2\right]$$

$$\text{FOR}\,\rho \approx 1 \Rightarrow \hat{x}_i = x_{i-1}$$

Linear Predictor

- Resulting Predictor:



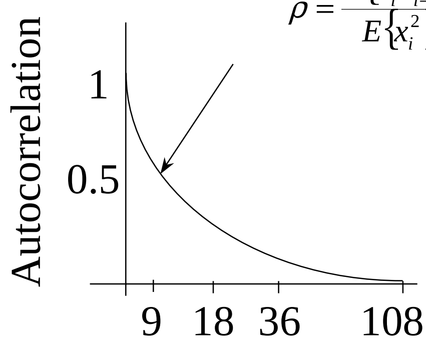
Prediction Error Δ

Differential Coder

Linear Predictor

- Correlation between the neighboring pixels

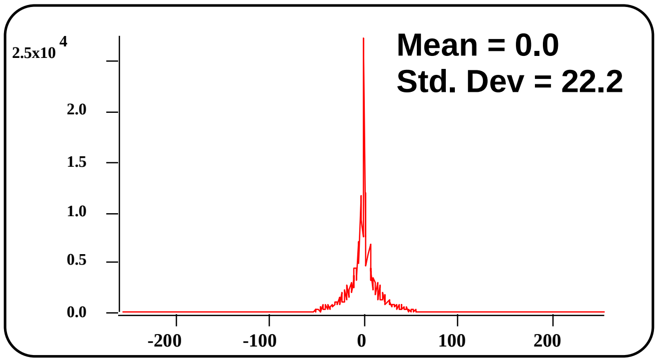
$$\rho = \frac{E\{x_i x_{i-1}\}}{E\{x_i^2\}}$$



Correlation between pixels decreases as distance between them increase

Predictor Error Probability Density

- The probability density function or histogram of prediction error is shown:



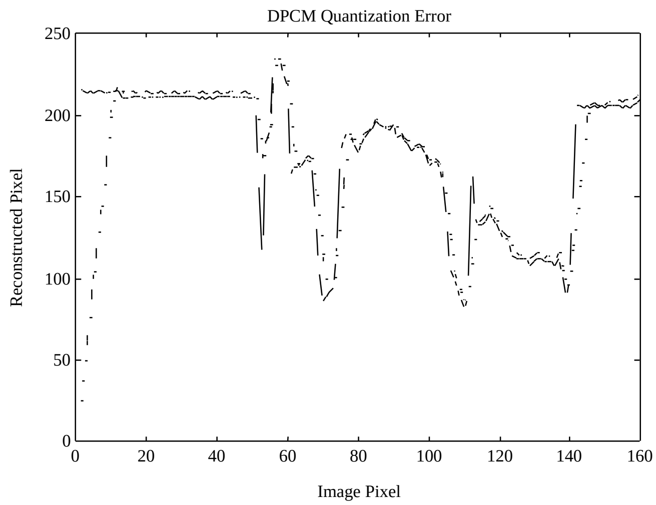
Lossy DPCM Error

DPCM quantizer based on Lloyd-Max 3 bit table from text

Data taken from first line of image.

Example of:

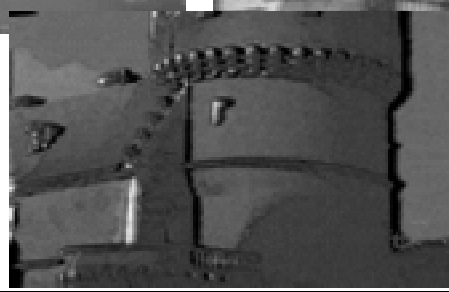
- Slope Overload
- Granular Noise
- Edge Busyness



Lossy DPCM Example

Original Image

DPCM Image



Difference Image

Lossy DPCM Results

- Image DPCM Lossy Compressed 1.3bpp
- Original Data 6.93 bpp
- Lossless DPCM rate: 2.67 bpp
- rmse = 10.84
- mse = 117.6

Minimum Mean Square Predictor

- A conditional mean predictor chooses

$$\hat{b}_N = \sum_{b_N} b_N P(b_N | b_1 \ldots b_{N-1})$$

which can be shown to minimize the mean square prediction error

$$E(b_N - \hat{b}_N)^2$$

where E is the statistical averaging or expectation operator.

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Minimum Mean Square Predictor

- A linear predictor chooses

$$\hat{b}_N = \sum_{i=1}^{N-1} \alpha_i b_i$$

where $\{b_i\}$ are the previous encoded pixels and $\{\alpha_i\}$ are the weighting coefficients that are chosen according to some performance criterion.

- For example, a "Maximum Likelihood" predictor chooses $\hat{b}_N$ to be that value of b_N that maximizes the conditional probability $P(b_N | b_1 \ldots b_{N-1})$

Minimum Mean Square Predictor

- A minimum mean square linear predictor can easily be derived by differentiating $E(b_N - \hat{b}_N)^2$ with respect to each α_j and setting the result to zero.
- This yields

$$\frac{\partial}{\partial \alpha_j}\left\{E\left(b_N - \sum_{i=1}^{N-1}\alpha_i b_i\right)^2\right\} = 0 \; for\; j = 1 \ldots M-1$$

$$-2E\left[b_N b_j - \sum_{i=1}^{N-1}\alpha_i b_i b_j\right] = 0$$

Minimum Mean Square Predictor

- Letting $d_j = E(b_N b_j)$ and $r_{ij} = E(b_i b_j)$ and using column matrices $D, \vec{\alpha}$ and square matrix R we get

$$D - R\vec{\alpha} = 0$$

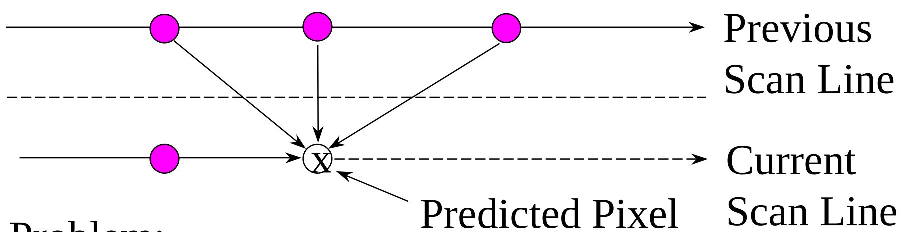
  The matrix R is known as the correlation matrix, and r_{ij} is the correlation between Pels b_i and b_j
- If the correlation matrix is nonsingular, then

$$\alpha = R^{-1}d$$

  Otherwise, many solutions exist.
  - Each having the same mean square prediction error: $E(b_N - \hat{b}_N)^2$

Other Linear Predictors

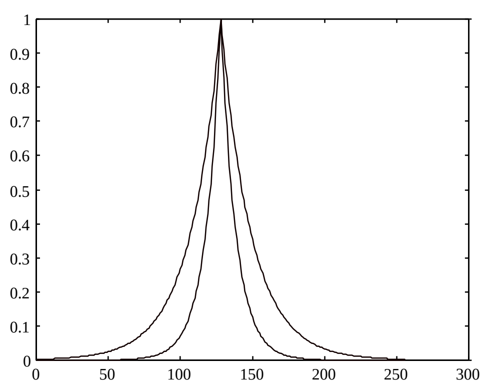
- 2-Dimensional Prediction



Problem:

Interlace in most systems reduces line-to-line correlation

Two Dimensional Predictor



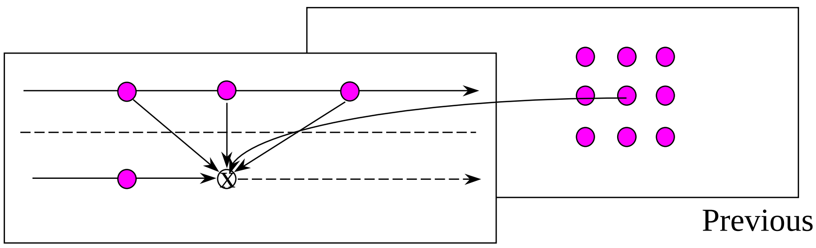
$$\hat{x}_{i,j} = \rho_1 x_{i-1,j-1} + \rho_2 x_{x-1,j} + \rho_3 x_{i-1,j+1}$$

Histogram of error signal for the one and two-D predictor

- Two Dimensional predictor produces smaller error signals than one dimensional predictor

Frame Difference Predictor

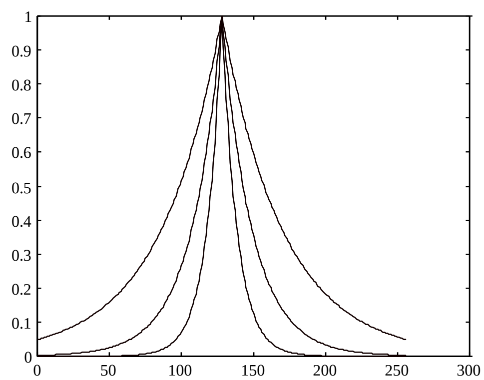
- For scenes with low detail and small motion, frame difference prediction appears to be the best.
- In scenes with higher detail and motion, intrafield prediction does better than frame prediction.



Current Frame

Previous Frame

3-D Prediction (Frame to Frame)

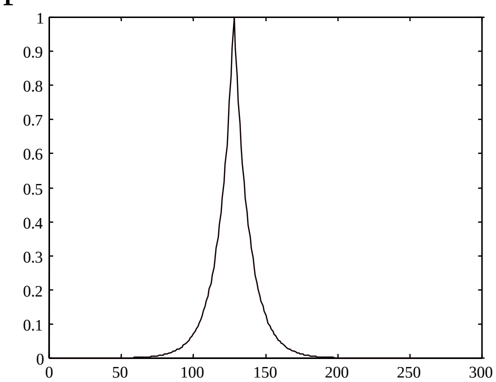


$$\hat{x}(i,j,t) = \rho_1 x(i,j,t-1) + \rho_2 x(i,j-1,t) + \rho_3 x(i-1,j,t) + \rho_4 x(i-1,j-1,t) + \rho_5 x(i-1,j+1,t)$$

- If there is no motion x(i,j,t) = x(i,j,t-1) but when there is a motion, an error signal is produced.
- Previous frame has to be stored

Statistics of Prediction Errors

- Histogram of error signal approximates an exponential distribution



$$P(\Delta) \approx \frac{k}{2} e^{-|k\Delta|}$$

Prediction Error Δ

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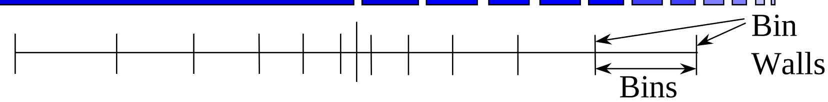
Non-Uniform Quantizer



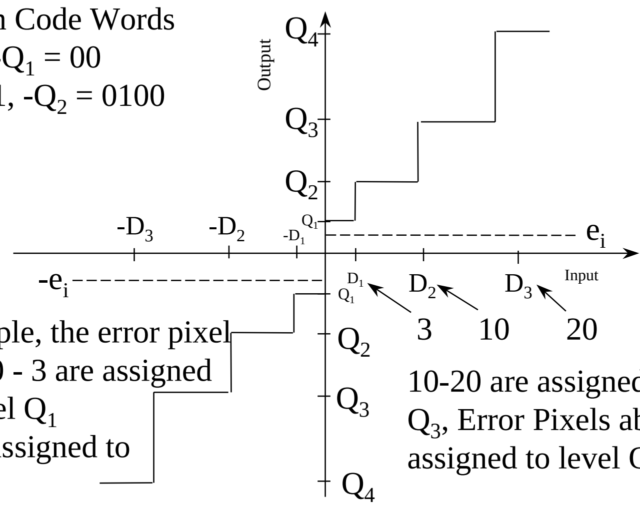
- Small errors for highly probable small D, large errors for unlikely, large D
  - Small average quantization error
- Large errors occur near sharp edges where eye has lowered sensitivity (Masking)
- Quantizer outputs tend to be equi-probable (Permits efficient, fixed length code)
- Edge Business

Quantizer Design For DPCM

- Consider an 8-level quantizer, the probability of occurrence of the quantized difference signals is not uniform, small differences are more numerous than large differences.
  - The output of the 8-level quantizer is coded by a Huffman Coder.
- In the 8-level quantizer, the levels are non-uniform

Quantizer Design For DPCM

Huffman Code Words
$Q_1 = 1$, $-Q_1 = 00$
$Q_2 = 011$, $-Q_2 = 0100$
$Q_3 =$



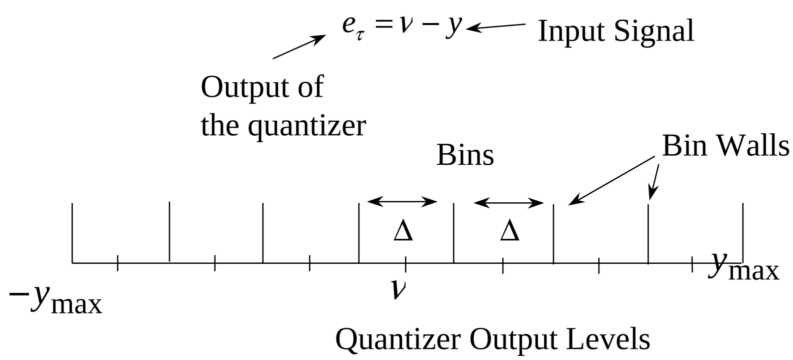
For example, the error pixel between 0 - 3 are assigned to the level Q_1
3-10 are assigned to level Q_2

10-20 are assigned to level Q_3, Error Pixels above 20 are assigned to level Q_4

Quantization Error

- The quantization error is the difference the quantizer output and input

$$e_\tau = v - y$$



Quantization Error

- Let Δ = bin width = $\dfrac{2y_{\max}}{2^R}$

$$\therefore \max e_\tau = \Delta/2$$

- The mean square error

$$e_\tau^2 = \int_{v-\Delta/2}^{v+\Delta/2} (y-v)^2 P(y)dy = \frac{\Delta^2}{12} = \frac{1}{3} y_{\max}^2 2^{-2R}$$

- If rms value of the input y is given by

$$y_{rms} = \sqrt{\int y^2 dy}$$

- Signal-to-Quantization noise ratio: $Q_{SNR} = \dfrac{y_{rms}}{e_{\tau_{rms}}}$
  or in dB $SNR = 10\log\dfrac{y_{rms}}{e_\tau}$